MerLean: An Agentic Framework For Autoformalization in Quantum Computation

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Abstract

We introduce MerLean, a fully automated agentic framework for autoformalization in quantum computation. MerLean extracts mathematical statements from LaTeX source files, formalizes them into verified Lean 4 code built on Mathlib, and translates the result back into human-readable LaTeX for semantic review. We evaluate MerLean on three theoretical quantum computing papers producing 2,050 Lean declarations from 114 statements in total. MerLean achieves full formalization without axioms on two of the three papers, while completing the third by introducing explicit axioms for results depending on mathematical machinery not yet available in Mathlib. Our results demonstrate that agentic autoformalization can scale to frontier research, offering both a practical tool for machine-verified peer review and a scalable engine for mining high-quality synthetic data to train future reasoning models. Our approach can also be generalized to any other rigorous research in mathematics and theoretical physics.

1 Introduction

The goal of autoformalization (Szegedy, 2020) is to automatically translate mathematical statements from natural language into formal code verifiable by systems such as Isabelle (Paulson, 1994), Coq (Team, 2024), and Lean (Moura & Ullrich, 2021). Recent work has shown that Large Language Models (LLMs) are well suited to this task due to their semantic processing capabilities (Wu et al., 2022; Jiang et al., 2022; Tarrach et al., 2024; Weng et al., 2025).

While most research in autoformalization focuses on pure mathematics, applications in physics have also attracted significant attention: Lean libraries such as PhysLean/HepLean (Tooby-Smith, 2024) and Lean-QuantumInfo (Meiburg et al., 2025) are currently under development, with the latter specifically utilizing autoformalization tools to accelerate the process. Theoretical quantum computation serves as an ideal testbed for this workflow. The volume of submissions to the arXiv `quant-ph` category reached 11,891 articles in 2025 (arXiv, 2025), creating a verification bottleneck that threatens to outpace the peer-review system. Functionally, the field operates as a subfield of applied mathematics, exhibiting deep connections to linear algebra, graph theory, and algebraic topology. The rigorous nature of these results makes them suitable for neuro-symbolic verification, while strong industrial interest ensures practical impact.

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In this work, we introduce **MerLean**, a fully automated agentic framework designed to extract, organize, and formalize mathematical statements from LaTeX source files, operating without human-in-the-loop intervention during the formalization process. After formalization succeeds, we run a second agent that "autoinformalizes" the verified code back into natural language, generating a blueprint for human experts to review the semantic alignment between the formal code and the original intent. We evaluate MerLean on three papers in theoretical quantum computing: one *unpublished* manuscript (guaranteeing zero data contamination) and two previously published works. MerLean achieved full formalization without axioms on both the unpublished manuscript and one published paper, while also completing the remaining paper by introducing necessary axioms for results depending on mathematical machinery not yet available in Mathlib. An interactive example of the formalization output is available at `https://doxtor6.github.io/MerLean-examples/`.

The scope of this framework extends beyond quantum computation. Any discipline that relies on formal mathematical proofs can use this neuro-symbolic approach to scale verification and accelerate discovery.

2 RELATED WORKS

LLM-based autoformalization and automated theorem provers demonstrated promising capabilities on competition-level problems (Wu et al., 2022; Jiang et al., 2022). Subsequent work improved upon these foundations by addressing proof detail expansion (Tarrach et al., 2024) and faithfulness verification (Li et al., 2024; Liu et al., 2025; Peng et al., 2025). Extensions to domain-specific applications include Euclidean geometry (Murphy et al., 2024), biomedical text (Zhang et al., 2025), and physics (Zhang et al., 2026).

Recent work has shifted toward agentic systems that couple frontier LLMs with tool integration for interactive theorem proving. Breen et al. (2025) demonstrated Ax-Prover, an MCP-based multi-agent system using Claude Sonnet that outperforms specialized provers on out-of-domain benchmarks. Xu & Odersky (2026) demonstrated "agentic proof automation" by mechanizing System Capless (14,000+ lines of Lean 4) with 87% task success rate across 189 annotated tasks; their workflow is *human-guided*, with humans providing definitions, theorems, and proof strategies while agents handle mechanical proof engineering. Liu et al. (2026) proposed Numina-Lean-Agent, combining Claude Code with MCP-based Lean tools including Lean-LSP-MCP for goal querying and theorem search; it solved all 12 Putnam 2025 problems without model training and formalized the Brascamp–Lieb theorem (8,000+ lines), though again requiring substantial human guidance.

Architecturally, MerLean closely resembles both systems—sharing the paradigm of agentic interaction with Lean 4 in a generate-check-refine loop, frontier LLMs as the reasoning backbone, and the ambition of formalizing research-level mathematics. The primary feature is that MerLean achieves full-paper and fully automated end-to-end formalization without human guidance on the domain we tested.

3 FRAMEWORK

MerLean is a bidirectional autoformalization framework comprising two complementary pipelines: *autoformalization* translates mathematical research papers from LaTeX into verified Lean 4 libraries built on Mathlib, while *autoinformalization* converts the formal code back into human-readable LaTeX for expert review. This round-trip design ensures both machine-verified correctness and human-verifiable semantic alignment. The framework employs a frontier LLM agent based on Claude Code that engages multi-turn agentic interactions—iteratively refining outputs based on compiler feedback, faithfulness checks, and tool-augmented exploration of Mathlib. Figure 1 illustrates the overall architecture.

3.1 AUTOFORMALIZATION

Statement Extraction. Given a LaTeX paper, the agent first extracts all mathematical *statements*, including definitions, theorems, lemmas, propositions, corollaries, and remarks into a structured JSON representation. Each statement corresponds to a single mathematical result from the paper. It includes a unique identifier (e.g., `Def_1`, `Thm_2`), the mathematical content in natural language,
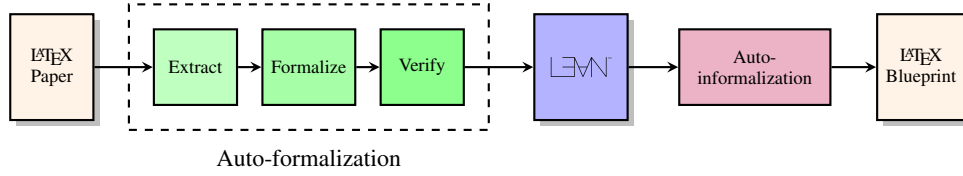
Figure 1: MerLean architecture. *Autoformalization* extracts statements from a LaTeX paper, formalizes them into Lean 4, and verifies faithfulness. *Autoinformalization* translates the verified code back into a human-readable LaTeX blueprint or file.

explicit dependencies on other statements, and proof sketches when available. The extraction runs for multiple iterations, where each pass reviews and refines the previous output: expanding vague phrases like "by standard arguments" into concrete proof steps, adding missing intermediate lemmas, and ensuring statements are ordered so that dependencies precede dependents.

Iterative Formalization. For each extracted statement, the agent enters a compile-fix loop. It generates Lean 4 code consisting of one or more *declarations*, writes them to the appropriate file in the library structure, and compiles it. Each statement typically produces multiple declarations, as the agent autonomously introduces auxiliary definitions, helper lemmas, and typeclass instances to support the target result. If compilation fails, the error messages (type mismatches, unknown identifiers, tactic failures, etc.) will be parsed and fed back to the agent, which analyzes the errors and produces a corrected version. This loop continues until either the code compiles without errors and warnings, or until it reaches a maximum attempt limit. During this process, the agent has access to several diagnostic tools provided via `lean-lsp-mcp`, a Model Context Protocol server that exposes Lean's language server capabilities directly to the agent, `lean_goal` for inspecting proof states at specific positions, `lean_hover_info` for type signatures and documentation, and semantic search tools (`leansearch`, `loogle`) for discovering relevant Mathlib lemmas. The agent can also grep through Mathlib source code to find usage patterns and verify that referenced lemmas actually exist.

Faithfulness Checking. Compilation alone is insufficient: an LLM can produce code that type-checks but misrepresents the mathematics (e.g., proving a trivial statement instead of the intended theorem) (Lin et al., 2025). After a successful build, MerLean reflects on whether the result matches the original meaning. If yes, the program continues to the next statement; otherwise it continues to attempt.

Axiom Handling. Frontier research often depends on results not yet in Mathlib (e.g., the Künneth formula for certain coefficient rings). MerLean handles this through explicit `axiom` declarations, clearly distinguishing intentional assumptions from incomplete proofs (`sorry`). When formalization fails after maximum attempts, an "axiom phase" converts blocking subgoals to axioms, producing partial formalizations with transparent assumptions that can be filled in as Mathlib expands.

3.2 AUTOINFORMALIZATION

The decoder reverses the formalization process, converting a verified Lean 4 library back into human-readable LaTeX. While existing tools such as `doc-gen4` can produce reasonably readable documentation from Lean source code, the output remains deeply tied to Lean's type-theoretic syntax and is difficult to interpret for readers without formal methods expertise. Therefore, MerLean utilizes the same LLM model that performed the formalization to do the informalization: since the model has demonstrated its ability to formalize content in the specific research area, it should also be capable of translating each declaration into natural language, making it accessible to domain experts with no prior knowledge of Lean. Of course, no original statement or paper content is provided to the informalization agent to prevent data leaks. The pipeline parses all Lean files, constructs a dependency graph, and produces two complementary outputs: an interactive blueprint compatible with `leanblueprint` for web-based exploration of the dependency structure, and a standalone textbook-style narrative for readers unfamiliar with formal methods. Any unverified assumptions (`axiom` declarations) are prominently highlighted, ensuring full transparency about the boundaries of the formalization.

4 EXPERIMENTS

We evaluate MerLean on three papers in theoretical quantum computing: one unpublished manuscript to guarantee zero data contamination, and two published papers to assess performance on existing literature.

- **Paper A: Balanced Product Codes (Breuckmann & Eberhardt, 2021).** This paper studies quantum codes constructed from tensor products and fiber bundles of chain complexes. The mathematical machinery includes homological algebra, tensor product of complexes, expander graphs and spectral expansion.
- **Paper B: Fault-Tolerant Quantum Computation (Williamson & Yoder, 2024).** This paper provides a comprehensive treatment of stabilizer codes and fault-tolerant protocols. The mathematical content includes stabilizer formalism and Pauli algebra, transversal gates, gauging graphs, fault-tolerant state preparation and measurement.
- **Paper C: Quantum Topology.** This is an unpublished manuscript, ensuring the content has never appeared in any LLM training data. The manuscript proves several algebraic and group-theoretic properties of some map on quantum computational systems. Details will be released later.

These three projects demonstrate the agent's capability to formalize and bridge logical gaps in frontier rigorous research, regardless of whether the content is present in the base LLM's training data. An interactive example of the Fault-Tolerant QC formalization output is available at `https://doxtor6.github.io/MerLean-examples/`.

4.1 RESULTS

Table 1: Summary of formalization experiments per paper.

Paper	Statements	Lines of Lean	Declarations	Time
Balanced Product	44	14,997	730	20h 4m
Fault-Tolerant QC	47	18,557	923	11h 41m
Quantum Topology	23	7,761	397	7h 51m

Table 2: Formalization statistics by statement type across all three papers.

Type	Count	Avg. Time	Avg. Compiles	Axiom Rate
Definition	49	18m 0s	11.7	0.0%
Theorem	15	39m 41s	22.4	20.0%
Lemma	20	33m 22s	18.3	5.0%
Remark	26	10m 34s	7.1	0.0%
Corollary	4	19m 23s	5.5	0.0%
Total/Avg	114	21m 54s	13.0	3.5%

Across all three papers, MerLean formalized 114 statements totaling 2,050 Lean declarations in under 42 hours of wall-clock time, consuming approximately 7.4M tokens. Both the unpublished Quantum Topology paper and the Fault-Tolerant QC paper achieved full formalization without axioms, demonstrating MerLean's capability on both novel and published content. The Balanced Product Codes paper required explicit axioms for 9.1% of statements, corresponding to results depending on machinery not yet in Mathlib (e.g., spectral sequences, Künneth isomorphisms for $\mathbb{F}_2$-chain complexes). Theorems were the hardest to formalize, averaging 39m 41s and 22.4 compile attempts with a 20.0% axiom rate, while remarks were the easiest at 10m 34s and 7.1 compiles with no axioms required. A representative fully-proved theorem is shown in Appendix A.1.

Figure 2 shows the distribution of compile attempts per statement, broken down by type and paper. The distribution is heavily right-skewed across all three papers: most statements compile within
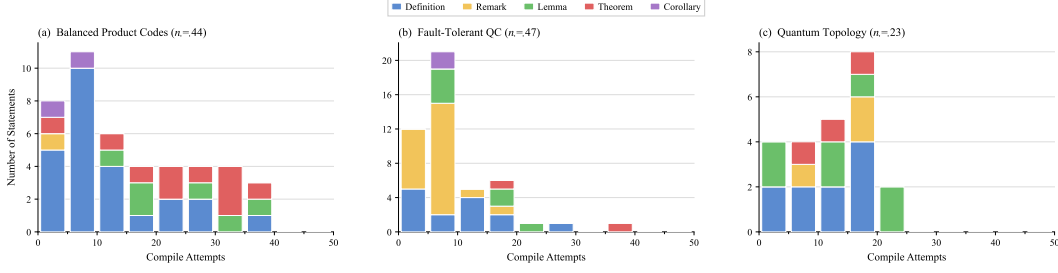
Figure 2: Distribution of compile attempts by statement type for each paper (maximum 30 attempts before axiom phase). Most statements resolve within 1–10 attempts; theorems and lemmas require significantly more iterations.

1–10 attempts, but a long tail of "hard" statements requires 20+ iterations. Balanced Product Codes exhibits the widest spread, reflecting its reliance on advanced algebraic machinery (spectral sequences, Künneth formulas) not yet in Mathlib. Fault-Tolerant QC is heavily concentrated in the low-compile regime, with 21 of 47 statements resolving within 5–10 attempts; its two theorems are the only statements exceeding 20 compiles. Quantum Topology shows a more uniform distribution, consistent with its smaller but technically diverse statement set. Across all papers, theorems and lemmas dominate the high-compile bins, while definitions and remarks cluster in the lower range.

4.2 QUALITATIVE OBSERVATIONS

Compile-Fix Loop Behavior. The hard statements that require 20+ compile attempts typically involve:

1. Dependent type arithmetic (e.g., proving $i - 1 + 1 = i$ at the type level)
2. Missing lemmas that require creative workarounds
3. Tactic timeouts requiring proof restructuring

Agent-Discovered Lemmas. MerLean frequently introduces auxiliary lemmas not stated in the original paper. These helper lemmas are natural intermediate steps that a human mathematician would typically leave implicit (see Appendix A.2 for a full example). For instance, the agent introduced:

- `edgeBoundary_card_eq_edgeCount` (*Balanced Product Codes*): Bridges the gap between the geometric definition of expansion (boundary size) and algebraic adjacency counts, enabling the formal derivation of the Relative Cheeger Inequality.

- `pauliPair_anticommuting_ct_satisfied` (*Fault-Tolerant QC*): Algebraically verifies that anticommuting spacetime faults cancel out disjointly in the detector model, a necessary intermediate result for proving the Global Correctness of Spacetime Syndrome Extraction.

- `weight_inequality_core` (*Fault-Tolerant QC*): Establishes that $w' - |S| + |\partial S| \geq \min(h(G), 1) \cdot d$ by case-splitting on the Cheeger parameter $h(G) \geq 1$, bridging the cleaning lemma and the space distance lower bound (see Appendix A.2).

Axiom Declarations. The Balanced Product Codes formalization required explicit axiom declarations where the underlying mathematical machinery is not yet available in Mathlib. Notably, the original paper invokes these classical theorems directly without proof, as they are well-known results in the literature; the informal proofs are therefore absent from the source material from the very beginning, making axiomatization the natural counterpart in the formal setting. These axioms reveal concrete gaps relevant to physics applications:

- **Künneth Formula**: The isomorphism $H_n(C \otimes D) \cong \bigoplus_{p+q=n} H_p(C) \otimes H_q(D)$ for chain complexes over $\mathbb{F}_2$ is not available in Mathlib (`kunnethMap_injective_aux`, `kunnethMap_surjective_aux`).

- **Tensor-Homology Commutativity**: The isomorphism $H_q(V \otimes C) \cong V \otimes H_q(C)$ for a flat module V is assumed via the combined axiom `e2PageIsoHomologyTensor_ax`. This conceptually bundles the vertical and horizontal homology steps to identifying the spectral sequence E^2 pages with tensor products of base and fiber homologies in Theorem 3. The separate axioms `verticalHomologyIsoTensor_ax` and `horizHomologyIsoTensor_ax` are defined but superseded by this combined form in the main proof.
- **Spectral Sequences**: The machinery for computing homology of total complexes via spectral sequences is incomplete (`spectralSequenceIsomorphism_nontrivial`). This is used in Theorem 3's proof of the Fiber Bundle Künneth Theorem to relate the E^2 page to the total homology.

These axioms are transparently marked in both the Lean code and the generated blueprint, so that readers immediately see which results rest on unverified assumptions. Appendix A.3 shows the full Künneth formula formalization as a concrete example.

5 DISCUSSION

5.1 CHALLENGES IN MERLEAN

Mathlib Gaps and Axiom-Based Formalization. Our evaluation highlighted gaps in Mathlib regarding specialized physics concepts, for instance, the Künneth formula was unavailable. MerLean handles this by explicitly declaring `axiom` nodes in the dependency graph. More broadly, many physics concepts are not rigorously defined in the language of mathematics (e.g., notions in quantum field theory). Yet, it is out of the scope of this work (and similar ones) to build lean code of full prior knowledge. Consequently, it is reasonable to develop libraries based on "axioms". Such choices of axioms should not be too elementary for the reason explained above, and not too advanced either, trivializing the derivation of the main results.

Faithfulness Checking. A critical challenge is ensuring that formalized code accurately reflects the original text, rather than merely compiling without errors. MerLean's faithfulness checking pipeline is used to address this problem, while the autoinformalized blueprint exposes the logical structure for human review. Together, these mechanisms eliminate hallucinations as much as possible, making it immediately apparent if the agent has proven a trivial variation or fabricated a definition to satisfy the compiler.

5.2 FUTURE APPLICATIONS OF MERLEAN

Research Assistant. During our initial formalization of the unpublished Quantum Topology paper, one lemma persistently required an axiom. Examining the autoinformalized blueprint revealed that an ambiguous definition, where a constraint the author had implicitly assumed was lacking, caused the problem. After fixing the LaTeX source, MerLean produced a complete formalization without axioms. This illustrates how the bidirectional pipeline can help researchers improve the rigor of their setups, definitions, and proofs.

Formalized Peer Review. Our validation on an unpublished manuscript confirms that MerLean operates without pre-training on the specific content. We envision a workflow where autoformalization occurs locally during drafting. Just as LaTeX became the standard for mathematical typesetting, agentic frameworks could make formal repositories a standard companion to static PDFs, transforming peer review dynamics: reviewers could rely on machine-verified guarantees, focusing strictly on novelty and scientific significance.

Synthetic Data Flywheel. Autoformalization also plays a critical role in training specialized theorem provers, as the quality of large-scale synthetic data depends directly on accurate formalization (Xin et al., 2024; Kumarappan et al., 2025). MerLean facilitates a virtuous cycle for LLM training by mining high-quality *(natural language, formal code)* pairs grounded in the scientific research they are based on. Feeding it back into base models will improve the next generation of agents.

Contributing to Physics Libraries. The Lean ecosystem has seen growing efforts to formalize physics, including PhysLean/HepLean (Tooby-Smith, 2024) for general physics and Lean-QuantumInfo (Meiburg et al., 2025) for quantum information theory. MerLean can accelerate contributions to these libraries by formalizing relevant research papers and extracting reusable definitions and lemmas. Alternatively, researchers can use MerLean to create domain-specific libraries by formalizing a collection of strongly related papers, building a coherent formal foundation for their research area that captures the interconnected definitions, lemmas, and theorems spanning multiple works.

6 CONCLUSION

We have presented MerLean, a fully automated bidirectional framework for formalizing mathematical research papers from LaTeX into verified Lean 4 libraries. Our evaluation on three papers in theoretical quantum computation, covering stabilizer codes, fault-tolerant protocols, balanced product codes, and homological algebra, produced over 2,000 Lean declarations across 41,000+ lines of verified code, demonstrating that fully automated formalization of frontier research is feasible. The iterative compile-fix-verify loop effectively produces faithful formalizations without human intervention, while the autoinformalization pipeline enables domain experts to review semantic alignment without formal methods expertise.

Looking ahead, while theoretical quantum computing serves as an effective testbed, its mathematical substrate, being primarily linear algebra and functional analysis, benefits from mature Mathlib support. We plan to extend our evaluation to other scientific domains and branches of pure mathematics characterized by deeper dependency chains or different foundational structures, such as algebraic geometry and number theory, to fully establish the framework's generalizability.

We also plan to evaluate MerLean on existing autoformalization benchmarks to enable direct comparison with prior work. However, current benchmarks (e.g., miniF2F, ProofNet) focus primarily on isolated theorem statements rather than full paper formalization with interconnected definitions, lemmas, and theorems. In the future, we intend to create a dedicated benchmark comprising diverse research papers across multiple mathematical domains, enabling systematic evaluation of autoformalization systems at the level of frontier research.

LLM USAGE DISCLOSURE

In accordance with ICLR 2026 policy, we disclose the following uses of large language models in this work:

Research. MerLean uses Claude (Opus 4.5) as its core reasoning engine for both autoformalization and autoinformalization. The LLM performs statement extraction from LaTeX, Lean code generation, error diagnosis and repair, and natural language translation of formal code. All experimental results reported in this paper were produced by this LLM-based system. The authors have verified and validated all research contributions.

Writing. Claude Code was used as a writing assistant for editing portions of this manuscript, including fixing typos and improving grammar. All content was reviewed, verified, and revised by the human authors, who take full responsibility for the accuracy and integrity of the final submission.

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A   FORMALIZATION EXAMPLES

We provide representative examples from the Fault-Tolerant QC formalization to illustrate the quality of MerLean's output.

A.1   THEOREM: FAULT TOLERANCE OF GAUGING MEASUREMENT

Theorem 2 of Williamson & Yoder (2024) (adapted): The fault-tolerant implementation of Algorithm 1 with a suitable graph G has spacetime fault-distance $d_{\mathrm{ST}} = d$, where d is the distance of the original code. Specifically, if (1) the Cheeger constant satisfies $h(G) \geq 1$, and (2) the number of measurement rounds satisfies $t_o - t_i \geq d$, then any spacetime logical fault has weight at least d.

```
/-- Main Theorem (Fault Tolerance): The spacetime fault-distance
    equals d. Under the conditions:
    1. h(G) >= 1 (Cheeger constant at least 1)
    2. (t_o - t_i) >= d (sufficient measurement rounds)
    The spacetime fault-distance d_ST equals exactly d. -/
theorem FaultToleranceTheorem
    [Fintype V] [Fintype E] [Fintype M] [Nonempty M]
    (DC : DetectorCollection V E M)
    (baseOutcomes : OutcomeAssignment M)
    (logicalEffect : SpacetimeFault V E M -> Prop)
    (cfg : FaultToleranceConfig)
    (h_all_decompose : forall F,
      IsSpacetimeLogicalFault DC baseOutcomes logicalEffect F ->
      LowerBoundCase DC baseOutcomes logicalEffect cfg F)
    (h_exists_d : exists F,
      IsSpacetimeLogicalFault DC baseOutcomes logicalEffect F
      /\ F.weight (intervalRounds cfg) = cfg.d) :
    spacetimeFaultDistance DC baseOutcomes logicalEffect
      (intervalRounds cfg) = cfg.d := by
```

```
    -- Get the weight-d logical fault for existence
    obtain <F_d, hF_d_log, hF_d_weight> := h_exists_d
    -- Upper bound: d_ST <= d
    have h_le : spacetimeFaultDistance DC baseOutcomes
        logicalEffect (intervalRounds cfg) <= cfg.d := by
      calc spacetimeFaultDistance DC baseOutcomes
            logicalEffect (intervalRounds cfg)
          <= F_d.weight (intervalRounds cfg) :=
            spacetimeFaultDistance_le_weight DC baseOutcomes
              logicalEffect (intervalRounds cfg) F_d hF_d_log
        _ = cfg.d := hF_d_weight
    -- Lower bound: d_ST >= d
    have h_ge : spacetimeFaultDistance DC baseOutcomes
        logicalEffect (intervalRounds cfg) >= cfg.d := by
      -- Get the minimum-achieving fault
      have h_has : hasLogicalFault DC baseOutcomes
        logicalEffect := <F_d, hF_d_log>
      obtain <F_min, hF_min_log, hF_min_weight> :=
        spacetimeFaultDistance_is_min DC baseOutcomes
          logicalEffect (intervalRounds cfg) h_has
      -- Apply lower bound to F_min
      have h_min_ge := spacetimeFaultDistance_ge_d DC
        baseOutcomes logicalEffect cfg F_min hF_min_log
        (h_all_decompose F_min hF_min_log)
      calc cfg.d
          <= F_min.weight (intervalRounds cfg) := h_min_ge
        _ = spacetimeFaultDistance DC baseOutcomes
            logicalEffect (intervalRounds cfg) := hF_min_weight
    -- Combine
    omega
```

A.2 AGENT-DISCOVERED HELPER LEMMA

This lemma is part of the formalization of Williamson & Yoder (2024), specifically addressing the lower bound on space distance. The agent introduced this intermediate lemma on its own to complete the proof of the space distance lower bound in the fault-tolerant quantum computation formalization. Given a Cheeger-like expansion parameter $h(G)$, a cleaning set of size at most the cleaned operator weight, and a boundary satisfying the isoperimetric inequality, the lemma establishes:

$$w' - |S| + |\partial S| \ \geq \ \min(h(G),\, 1) \cdot d,$$

where w' is the cleaned weight, $|S|$ the cleaning set size, $|\partial S|$ the boundary size, and d the code distance. The proof splits on whether $h(G) \geq 1$, using a calc chain with nlinarith in each branch.

```
theorem weight_inequality_core
    (hG : R) (hG_nonneg : 0 <= hG)
    (d : N) (_hd_pos : 0 < d)
    (cleanedWeight : N)
    (hCleaned : cleanedWeight >= d)
    (cleaningSetSize : N)
    (hCleaningBound : cleaningSetSize <= cleanedWeight)
    (boundarySize : N)
    (hCheeger : (boundarySize : R) >= hG * cleaningSetSize) :
    (cleanedWeight : R) - cleaningSetSize + boundarySize
      >= minCheegerOne hG * d := by
  simp only [minCheegerOne]
  by_cases hG_ge_1 : hG >= 1
  . -- Case h(G) >= 1: boundarySize >= cleaningSetSize, so result
      >= cleanedWeight >= d
```

```
      rw [min_eq_right hG_ge_1]
      have hBound : (boundarySize : R) >= cleaningSetSize := by
        calc (boundarySize : R) >= hG * cleaningSetSize := hCheeger
          _ >= 1 * cleaningSetSize := by nlinarith
          _ = cleaningSetSize := one_mul _
      have hCleaned' : (cleanedWeight : R) >= d :=
        Nat.cast_le.mpr hCleaned
      linarith
    . -- Case h(G) < 1
      push_neg at hG_ge_1
      rw [min_eq_left (le_of_lt hG_ge_1)]
      have hClean : (cleanedWeight : R) >= d :=
        Nat.cast_le.mpr hCleaned
      have hS' : (cleaningSetSize : R) <= cleanedWeight :=
        Nat.cast_le.mpr hCleaningBound
      calc (cleanedWeight : R) - cleaningSetSize + boundarySize
          >= cleanedWeight - cleaningSetSize
            + hG * cleaningSetSize := by linarith
        _ = cleanedWeight
            + (hG - 1) * cleaningSetSize := by ring
        _ >= cleanedWeight
            + (hG - 1) * cleanedWeight := by nlinarith
        _ = hG * cleanedWeight := by ring
        _ >= hG * d := by nlinarith
```

A.3 AXIOM EXAMPLE: KÜNNETH FORMULA

This example is drawn from the formalization of Breuckmann & Eberhardt (2021), where the Künneth formula is essential for calculating the homology of balanced product codes. The Künneth formula is a fundamental result in homological algebra that relates the homology of a tensor product of two chain complexes to the tensor products of their individual homologies. Given chain complexes $C = (C_\bullet, \partial^C)$ and $D = (D_\bullet, \partial^D)$ over $\mathbb{F}_2$, the formula states:

$$H_n(C \otimes D) \cong \bigoplus_{p+q=n} H_p(C) \otimes H_q(D).$$

Over a field such as $\mathbb{F}_2$, all modules are flat, so the Tor correction terms vanish and the Künneth map is an isomorphism. The isomorphism sends a pair of homology classes $[z] \in H_p(C)$ and $[w] \in H_q(D)$ to the class $[z \otimes w] \in H_{p+q}(C \otimes D)$. Proving this in Lean 4 requires (i) showing the tensor product of cycles is a cycle, (ii) showing boundaries are preserved so the map descends to homology, and (iii) establishing bijectivity. Steps (i)–(iii) involve unfolding the internal colimit structure of Mathlib's total complex, which is not yet supported; these are axiomatized. The full formalization is shown below.

The 4 snippets below correspond to the definition of types, followed by the three proof steps (i)–(iii).

First, we define the source type (direct sum of tensor products of homologies) and the target type (homology of the tensor product):

```
variable (C D : F2ChainComplex)

/-- All F2-modules are flat (free modules are flat). -/
instance flat_F2_module (M : Type*) [AddCommGroup M]
    [Module F2 M] : Module.Flat F2 M :=
  Module.Flat.of_free

/-- Tensor product of homology spaces. -/
noncomputable def HomologyTensor (p q : Z) : Type :=
  (C.Homology p) (x)[F2] (D.Homology q)
```

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```
...

/-- Index set: pairs (p,q) with p + q = n. -/
def KunnethIndex (n : Z) : Type :=
  { pq : Z x Z // pq.1 + pq.2 = n }

/-- The direct sum (+)_{p+q=n} H_p(C) (x) H_q(D). -/
noncomputable def KunnethDirectSum (n : Z) : Type :=
  Pi_0 (i : KunnethIndex n),
    HomologyTensor C D i.val.1 i.val.2

...

/-- Homology of the tensor product complex at degree n. -/
noncomputable abbrev TensorHomology (n : Z) : Type :=
  (TensorProductComplex C D).Homology n
```

Step (i): Constructing the map on cycles. We show that the tensor product of cycles is a cycle, allowing us to define the cross product on cycles:

```
/-- Map from C_p (x) D_q to (C (x) D)_{p+q}. -/
noncomputable def tensorInclusion (p q : Z) :
    (C.X p) (x)[F2] (D.X q) ->l[F2]
      (TensorProductComplex C D).X (p + q) :=
  (TensorProductComplex.i C D p q).hom

/-- Axiom: z (x) w is a cycle when both z, w are cycles.
d(z (x) w) = dz (x) w +- z (x) dw = 0. -/
axiom cycle_tensor_cycle_is_cycle'_aux
    (C D : F2ChainComplex) (p q : Z)
    (z : C.X p) (hz : z in C.Cycles p)
    (w : D.X q) (hw : w in D.Cycles q) :
    tensorInclusion C D p q (z (xt) w) in
      (TensorProductComplex C D).Cycles (p + q)

/-- Map from C.Cycles p (x) D.Cycles q to total cycles. -/
noncomputable def cyclesCrossProduct (p q : Z) :
    (C.Cycles p) (x)[F2] (D.Cycles q) ->l[F2]
      (TensorProductComplex C D).Cycles (p + q) := by
  ...  -- bilinear map lifting cycle_tensor_cycle_is_cycle'
```

Step (ii): Descent to homology. We axiomatize that boundaries are preserved under the cross product, ensuring the map descends to a well-defined map on homology:

```
/-- Axiom: boundary (x) cycle is a boundary.
If z = dz', then z (x) w = d(z' (x) w) since dw = 0. -/
axiom boundary_tensor_cycle_is_boundary'_aux
    (C D : F2ChainComplex) (p q : Z)
    (z : C.Cycles p) (hz : z.val in C.Boundaries p)
    (w : D.Cycles q) :
    (cyclesCrossProduct C D p q (z (xt) w)).val in
      (TensorProductComplex C D).Boundaries (p + q)

/-- Axiom: cycle (x) boundary is a boundary.
If w = dw', then z (x) w = d(z (x) w') since dz = 0. -/
axiom cycle_tensor_boundary_is_boundary'_aux
    (C D : F2ChainComplex) (p q : Z)
    (z : C.Cycles p)
```

```
              (w : D.Cycles q) (hw : w.val in D.Boundaries q) :
              (cyclesCrossProduct C D p q (z (xt) w)).val in
                (TensorProductComplex C D).Boundaries (p + q)
```

Step (iii): Bijectivity and the Main Theorem. Finally, we construct the Künneth map and axiomatize
its bijectivity to establish the isomorphism:

```
  ...

  /-- Cross product descends to homology: [z] (x) [w] |-> [z (x) w].
      -/
  noncomputable def kunnethComponentMap (p q : Z) :
      HomologyTensor C D p q ->l[F2]
        TensorHomology C D (p + q) :=
    TensorProduct.lift (kunnethComponentMapAux2 C D p q)

  /-- The Kunneth map from (+)_{p+q=n} H_p(C) (x) H_q(D)
  to H_n(C (x) D). -/
  noncomputable def kunnethMap (n : Z) :
      KunnethDirectSum C D n ->l[F2]
        TensorHomology C D n := by
    refine DFinsupp.lsum N ?_
    intro <<p, q>, hpq>
    subst hpq
    exact kunnethComponentMap C D p q

  /-- Axiom: the Kunneth map is injective over F2. -/
  axiom kunnethMap_injective_aux
      (C D : F2ChainComplex) (n : Z) :
      Function.Injective (kunnethMap C D n)

  /-- Axiom: the Kunneth map is surjective over F2. -/
  axiom kunnethMap_surjective_aux
      (C D : F2ChainComplex) (n : Z) :
      Function.Surjective (kunnethMap C D n)

  /-- Kunneth Formula:
  (+)_{p+q=n} H_p(C) (x) H_q(D) ~= H_n(C (x) D). -/
  noncomputable def kunnethEquiv (n : Z) :
      KunnethDirectSum C D n ~=l[F2]
        TensorHomology C D n :=
    LinearEquiv.ofBijective (kunnethMap C D n)
      <kunnethMap_injective C D n,
       kunnethMap_surjective C D n>
```